Name:________________________

MA 3113 Section 51 *Practice Exam 2* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit! On the actual exam the last page will have a list of matrices and their row reduced echelon or row echelon forms that you may or may not need.

1. Let
$$C^1(\mathbb{R}) = \{f(x) : f'(x) \text{ exists and is continuous}\}$$
Show that $C^1(\mathbb{R})$ is a subspace of $C(\mathbb{R})$, the set of all continuous functions on the real line under the usual addition of functions and scalar multiplication.

2. Let
$$C_a[a, b] = \{f(x) : f(a) = 0\}$$
Show that $C_a[a, b]$ is a subspace of $C[a, b]$, the set of all continuous functions on the interval $[a, b]$ under the usual addition of functions and scalar multiplication.

3. Let
$$S = \left\{t^2 + 1, t^2 + t, t + 1\right\}$$
Determine if the following set is a basis for P_2, the set of all polynomials of degree 2 or less.

4. Let
$$S = \left\{2t^2 + 1, 3t^2 - 4t, 4t + 9\right\}$$
Determine if the following set is a basis for P_2, the set of all polynomials of degree 2 or less.

5. Let $T : P_2 \to \mathbb{R}$ be a linear transformation defined by:

$$T(p(t)) = \int_0^1 (a_0 + a_1 t + a_2 t^2)\ dt$$

Compute Ker(T).

6. Let $T : P_2 \to P_1$ be a linear transformation defined by:

$$T(p(t)) = \frac{d}{dt}(a_0 + a_1 t + a_2 t^2)$$

Compute Ker(T).

7. Let A be an 10×10 matrix and let $d = \dim \mathrm{Ker}(A)$. Suppose d solves the following quadratic equation:

$$d^2 - 9d = 0$$

What are the possible dimensions of the columnspace to A? In either case is A invertible? (HINT: You will need the rank-nullity theorem)

8. Let A be an $m \times n$ matrix. Let $A^t A$ be nonsingular. Show that $\mathrm{rk}(A) = n$. (HINT: You will need the rank-nullity theorem)

9. Use determinants to determine if the following matrix is invertible:

$$A = \begin{pmatrix} 2 & 6 & 0 \\ 1 & 3 & 2 \\ 3 & 9 & 2 \end{pmatrix}.$$

10. Use determinants to determine if the following matrix is invertible:

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 8 & 8 & 8 \end{pmatrix}.$$

11. Suppose A is a square matrix such that $\det(A^3) = 0$. Prove that A cannot be invertible.

12. Let A and P be square matrices with P invertible. Show that $\det(PAP^{-1}) = \det(A)$.

13. Let A be an $n \times n$ matrix. Suppose there is no nonzero vector $\mathbf{x} \in \mathbb{R}^n$, such that $A\mathbf{x} = \mathbf{x}$. Show that $A - I_n$ is nonsingular. (HINT: You will need the rank-nullity theorem.)